Multi-Product Bargaining, Bundling, and Buyer Power: Online Appendix*

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Proofs

Proof of Lemma 1: We prove by contradiction that one of the manufacturer's constraints has to bind if the retailer's buyer power becomes sufficiently strong. Let the manufacturer's bargaining power α' be non-zero, but sufficiently small so that the retailer's constraint is not binding. By the proof of Lemma 2 in OBS, such an $\alpha' > 0$ exists.

Now suppose that one of the manufacturer's constraints (8) or (9) does not bind. Without loss of generality, assume that constraint (8) on the incentive to manufacture product 2 in addition to product 1 does not bind. Then, the first-order condition of the Nash product with respect to F_2 has to be satisfied: that is,

$$F_1 + F_2 = \alpha R(q_1, q_2) + (1 - \alpha)C(q_1, q_2),$$

so that

$$\lim_{\alpha \to 0} F_1 + F_2 = C(q_1, q_2).$$

The resulting equilibrium quantities q_1, q_2 and transfers F_1, F_2 all depend on α.

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Due to our assumption on the cost function, (1), for any small α there is some $\epsilon'(\alpha) > 0$ so that $C(q_1, q_2) > C(q_1, 0) + C(0, q_2) + \epsilon'(\alpha)$ as for any α both equilibrium quantities q_1, q_2 are strictly positive. Let $\epsilon'' := \min_{\alpha \in [0,\alpha']} \epsilon'(\alpha)$. Next, for ϵ'' (which is strictly positive) there is some $\alpha'' \in (0,1)$ so that for all $\alpha < \alpha''$ we have

$$\epsilon'' > F_1 + F_2 - C(q_1, q_2).$$

From the manufacturer's constraints $F_1 + F_2 - C(q_1, q_2) \geq F_1 - C(q_1, 0)$ and $F_1 + F_2 - C(q_1, q_2) \geq F_2 - C(0, q_2)$, we obtain $F_1 + F_2 \geq 2C(q_1, q_2) - C(q_1, 0) - C(0, q_2)$. Hence, for all $\alpha < \alpha''$ we have

$$\epsilon'' > F_1 + F_2 - C(q_1, q_2) \geq C(q_1, q_2) - C(q_1, 0) - C(0, q_2) > \epsilon'',$$

which is a contradiction. Thus, if α is sufficiently small, the manufacturer's constraints (8) and (9) must bind. □

Proof of Proposition 2: Suppose $\alpha < \underline{\alpha}$. We distinguish two cases depending on whether or not the conditions imposed in Lemma 1 hold.

Case 1. First assume that the conditions imposed for Lemma 1 hold. We show by contradiction that in this case quantities are downward distorted.

(A) Assume that quantities are upward distorted, $q_1 \geq q_1^I$ and $q_2 \geq q_2^I$, where one of the inequalities is strict. Fix F_1 and F_2, but marginally decrease the output quantities in the direction of their efficient levels. This relaxes incentive constraints (8) and (9) due to cost substitutability. Note that q_1 and q_2 cannot be an (interior) solution of the respective maximization problem as this is assumed to be unique at (q_1^I, q_2^I). By Pareto optimality of the Nash bargaining solution, q_1 and q_2 cannot be an equilibrium outcome.

(B) Next suppose that one quantity is up- and one quantity is downward distorted so that without loss of generality $q_1 > q_1^I$ and $q_2 < q_2^I$ holds. Fix F_1 and F_2, but decrease q_1 and slightly increase q_2, so that $C(q_1, 0)$ and $C(q_1, q_2)$ both decrease while $C(q_1, 0)$ decreases by less than $C(q_1, q_2)$. This is possible due to cost substitutability and makes the manufacturer's constraints slack. As in (A), q_1 and q_2 cannot be an (interior) solution of the respective maximization problem as this is assumed to be unique at (q_1^I, q_2^I). By Pareto optimality of the Nash bargaining solution, q_1 and q_2 cannot be an equilibrium outcome.

(C) Third, assume that $q_i < q_i^I$ and $q_j = q_j^I$ where $i \neq j$. A marginal reduction of q_j has only a second-order negative effect, while a marginal increase of q_i has a first-order positive effect on joint profits. A marginal increase of F_i , and a marginal reduction of F_j will give these quantity effects. By Pareto optimality, therefore, $q_i < q_i^I$ and $q_j = q_j^I$ cannot constitute an equilibrium. Analogously, $q_i > q_i^I$ and $q_j = q_j^I$ cannot constitute an equilibrium either.

Case 2. Finally suppose that Lemma 1 does not apply as one or two of the retailer's incentive constraints strictly bind. If (8) and (9) do not bind, then, by OBS, equilibrium quantities are below their efficient levels. If (8) or (9) bind, equilibrium quantities are downward distorted likewise due to Pareto optimality of the Nash product and cost substitutability (arguments are analogous to those in our preceding analysis).

Thus, in equilibrium both quantities are downward distorted under our initial assumption that both products are provided in strictly positive quantities. □