Third-Degree Price Discrimination in Oligopoly When Markets Are Covered*

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May 2021

Abstract

We analyze oligopolistic third-degree price discrimination relative to uniform pricing when markets are covered. Pricing equilibria are critically determined by supply-side features such as the number of firms and their marginal cost differences. It follows that each firm's Lerner index under uniform pricing is equal to the weighted harmonic mean of the firm's relative margins under discriminatory pricing. Uniform pricing then decreases average prices and raises consumer surplus. We provide an intriguingly simple approach to calculate the gain in consumer surplus and loss in firms' profits from uniform pricing only based on market data of the discriminatory equilibrium (prices and quantities).

JEL-Classification: D43, L13, L41, K21.

Keywords: Third-Degree Price Discrimination, Oligopolistic Competition, Market Integration.

*We thank Takanore Adachi, Levent Celik, Mark Armstrong, Mats Köster, and Markus Reisinger for very helpful comments. Christian Wey gratefully acknowledges financial support by the German Science Foundation (DFG) for the Research Training Group "Competition Economics" (GRK 1974).

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1 Introduction

1.1 Motivation and Contribution

Third-degree price discrimination is a key management practice, for instance, in the form of geographic market segmentation, whereby different retail prices are charged in different countries of the European Union.[1] Due to its explicit policy objective to create a *Single Market*, the European Union, however, has recently passed a geo-blocking directive (EU Regulation 2018/302) that prevents such discriminatory pricing practices at least in parts, that is, for online stores that discriminated final consumers on the grounds of their geographic (i.e., country) location. This directive has spurred an ongoing policy debate whether geo-blocking and corresponding market segmentation practices[2] which support discriminatory practices are to be banned or not.[3] While much is known now about the welfare effects and the profitability of third-degree price discrimination in a monopoly, our knowledge about oligopolistic third-degree price discrimination is less clear-cut. Based on the logic of the widely used Hotelling model of product differentiation, we show that under price discrimination prices in markets with low competitive intensities increase by more than prices in markets with high competitive intensities decline, as a consequence of which preventing discriminatory practices (for instance by banning geo-blocking) is undoubtedly benefiting consumers.

In detail, our contribution is to analyze the welfare effects and the profitability of third-degree price discrimination in an oligopoly, where firms have different marginal production costs. We first show that firms' (bilateral) price differences in any market are always the same, independently of the pricing regime. Price differences only depend on supply-side features and

[1]See ECB (2011) for a documentation of substantial price differences of international brands (in the food industry) across Euro-area countries.

[2]Recently, the European Commission has fined *AB InBev*, the world's largest beer brewer, for implementing "territorial supply constraints" which facilitated price discrimination across countries by preventing cross-border sales at the wholesale level. The Commission declared such discrimination by geographical markets as incompatible with the rules of the Single Market (see the Commission Decision C(2019) 3465, case AT.40134 – AB InBev beer trade restrictions).

[3]See, e.g., `https://www.europarl.europa.eu/thinktank/en/search.html?word=geo-blocking&page=1` (accessed on 6 May, 2021). .

are independent of the parameters of the demands. It then follows that firms' output levels are the same under discriminatory and uniform pricing in all markets. As a consequence of this, price discrimination does not affect social welfare. Nevertheless, firms' demands and market shares across markets may differ depending on competitive intensities which in turn depend on demand parameters. Our second finding is based on this result and it comes in two practically important formulas: Firstly, we show that each firm's aggregate price elasticity under uniform pricing is the weighted arithmetic mean of the firm's market-specific price elasticities under discriminatory pricing, where the weights are given by the firm's output in market j relative to its total output. Secondly, the relative margin (or, Lerner index) under uniform pricing is given by the weighted harmonic mean of the firm's relative margins (or, Lerner indices) under discriminatory pricing, where the weights are given again by the firm's output in market j relative to its total output.

The harmonic mean logic implies that the relative margin under uniform pricing is always strictly lower than the weighted arithmetic mean of the relative margins under discriminatory pricing; in other words, market power is reduced. This translates into the aggregate Lerner index being smaller under uniform than under discriminatory pricing. There is, unambiguously, a consumer surplus loss from price discrimination, which can be easily calculated only based on observables under discriminatory pricing. Simply from market prices and firms' outputs we can recover consumer surplus under uniform pricing.

Due to best-response symmetry—whereby firms agree in which market segment to set the higher and where the lower prices—firms have clear incentives to collectively achieve the price discrimination outcome. For instance, firms may want to segment markets and prevent arbitrage to make price discrimination possible. From the firms' perspective, the discriminatory equilibrium represents a Pareto-improvement vis-à-vis the equilibrium under uniform pricing. Notably, all our results also apply when firms have market-specific delivery costs and engage in *differential pricing*.

Our demand system is closely related to the one proposed by Somaini and Einav (2013), who derived it from generalizing the Hotelling duopoly model to the case of $m \geq 2$ firms. Demand is always covered, all firms are directly linked and compete this way symmetrically with each other. In analogy to the monopoly benchmark, which exclusively highlights the demand-side determinants of the welfare effects of price discrimination, we analyze the oligopoly case with inelastic market demands to focus the analysis on the supply-side determinants of price discrimination and its welfare effects. We achieve this modeling approach with a "covered demand" model, which we do not regard it as implausible: when for a particular product the *relevant market* is considered, all products consumers can substitute to are already included, so that the assumption that market demand is fully price inelastic is plausible under this practice. And indeed, antitrust authorities typically define the market to be considered as the relevant market—as delineated by the SSNIP ("Small but Significant and Non-transitory Increase in Price") test—that comprises all substitutes to a particular product up to a certain threshold. Importantly, even if with covered demand price discrimination has no effect on social welfare, it affects consumer surplus, which represents the objective of most antitrust authorities (see, e.g., Davies and Lions, 2007, or Whinston, 2007). Finally, our demand system allows for much flexibility: demand characteristics affect the size of the different markets, the price levels, and firm's market shares that can vary across markets.

In an extension, we show that our insights also hold if price discrimination is constrained by arbitrage. Practically, unconstrained price discrimination can only become effective if arbitrageurs cannot resell goods sourced in the low-price region to the high-price region (see Armstrong, 2008). Thus, when policy makers wish to discourage price discrimination, they will often take the indirect route of ensuring that consumer arbitrage is as easy as possible, for instance by integrating markets (see Armstrong, 2008). In the EU, the creation of a *Single Market* is an explicit policy objective. Accordingly, the European Union has passed the geo-blocking directive (EU Regulation 2018/302), which bans price discrimination of online stores vis-à-vis final consumers on the grounds of their geographic (i.e., country) location since 2018. This recent geo-blocking directive is fitting this strategy as it tries to enhance cross-border arbitrage by consumers. If markets are perfectly integrated in the sense that consumers can buy a certain good in any other country at the terms posted in that country, then any international price discrimination is doomed to fail, so that the products of any firm i must be traded at the same price in the integrated market area. By our analysis, such market integration—which makes arbitrage as easy as possible and effectively yields uniform pricing—is desirable from a consumer point of view.

1.2 Related Literature

The related literature can be divided into the literature on monopolistic and oligopolistic third-degree price discrimination. The literature on monopolistic third-degree price discrimination has focused on the demand conditions which determine the welfare effects of price discrimination. This welfare effect results from a trade-off between the misallocation effect and the output effect relative to the uniform pricing rule. While Pigou considered the linear (downward sloping) demand case, Robinson (1933) and Schmalensee (1981), and more recently Aguirre, Cowan and Vickers (2010) derived complementary results for convex and concave demands. Varian (1985) extends Schmalensee (1981) by allowing for imperfect arbitrage when marginal costs are constant or increasing, and Schwartz (1990) extends Varian (1985) for the case where marginal costs are decreasing. Cowan (2012, 2016) focuses on the social welfare and consumer surplus effects of monopolistic third-degree price discrimination depending on market demands. He identifies, beside other things, "reasonable" demand conditions such that price discrimination increases consumer surplus. The main insight from this literature is that when switching from uniform pricing to third-degree price discrimination the price rises in one (the "strong") and falls in the other (the "weak") market, and the curvatures of demand functions are critical for the resulting welfare effect. It remains an open question, however, in how far these insights apply to oligopolistic markets.

The literature on oligopolistic third-degree price discrimination is relatively sparse. It has to be divided into approaches that build on best-response symmetry—where firms agree on where to set higher prices—and those that build on best-response asymmetry—where firms disagree on where to set higher prices (see Corts 1998). Under best-response asymmetry, firms disagree where to set higher and where to set lower prices; in this case, firms find themselves in a prisoners dilemma as price discrimination intensifies competition (see, e.g., Thisse and Vives 1988; Armstrong 2008). Firms then have a collective incentive to prevent price discrimination (see, e.g., Stole 2007). The literature on best-response symmetry started out with Holmes (1989), who mainly showed that the output effect of third degree price discrimination is the sum of

Schmalensee's (1981) adjusted concavity condition (which mirrors the market demand effect) and the elasticity-ratio condition (which picks up the oligopolistic competition effect).[4] Subsequent work on oligopolistic third-degree price discrimination with symmetric firms has been further studied in Armstrong and Vickers (2001), Weyl and Fabinger (2013), Adachi and Fabinger (2020) and Miklos-Thal and Shaffer (2021). Armstrong and Vickers (2001), in particular, show that for sufficiently competitive markets, price discrimination increases profits and reduces welfare. Tan and Zhou (2021) analyze oligopolistic price competition with multi-sided markets and a general discrete choice demand, and as a side finding (see their Proposition 7 in Section C of the Appendix), they show for the case where firms have identical costs that engaging in third-degree price discrimination is more profitable than uniform pricing. Building on earlier work for the monopolistic case (Chen and Schwartz, 2015), Chen et al. (2021) analyze differential pricing in oligopolies where market-delivery costs differ across markets. With such market-specific delivery costs, uniform pricing necessarily induces an allocative inefficiency as cost differences cannot be reflected in prices; our main insights extend to the case of market-delivery costs. As all our results also apply when firms have market-specific delivery costs, we also contribute to this strand of the literature.

Finally, Adachi and Fabinger (2020) extend Aguirre, Cowan, and Vickers (2010) to the oligopoly case for symmetric firms case.[5] Altogether, the existing literature does not determine the welfare effects of third-degree price discrimination in asymmetric oligopolies.

We proceed as follows. In Section 2 we analyze the covered market model. Section 3 discusses our extension on arbitrage costs. Finally, Section 4 concludes.

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[4]In a spatial model of monopolistic competition that is not analytically tractable, Borenstein (1985) compares different sorting criteria for firms, and finds, using computer simulations, that price discrimination based on consumers' reservation prices tends to better for consumers than discrimination based on consumers' strength of brand preferences.

[5]In the last part of their paper, Adachi and Fabinger (2020) also discuss heterogeneous firms (see their Proposition 7), but do not derive results based on exogenous model inputs.

2 Model

2.1 The LCD-Model

We build on the (linear-) covered demand model (in short: LCD-model), which is closely related to the generalized Hotelling model proposed by Somaini and Einav (2013). Assume m firms sell their products in n markets, where $m, n > 1$. Each firm produces a single product and firm i's marginal production cost is $c_i \geq 0$.

Market demands are independent and completely inelastic. The demand of firm i in market j is a linear function of its own price and all other firms' prices in that market. We assume symmetry in all substitutability relations. In addition, all products are directly linked, so that consumers as a whole can substitute away to all other products. Taken together, we obtain a (linear-) covered demand model $LCD := \{D_i^j\}_{i=1,...,m}^{j=1,...,n}$, where the demand of firm i in market j is given by

$$D_i^j(p_1^j, ..., p_m^j) = a^j + b^j \sum_{i' \neq i}(p_{i'}^j - p_i^j), \text{ with } a^j > 0 \text{ and } b^j > 0. \quad (1)$$

The LCD-model nests the Hotelling duopoly model and the Salop-circle model for two and three firms.[6] It does not nest the Salop model for four and more firms. To understand the difference, take $m = 4$. In the Salop model each firm only competes directly with its two neighbors and not with the remaining competitor. This kind of asymmetry of the Salop model is eliminated in our model, where all firms compete directly. In the LCD-model the four firm case can be thought of represented by six equally long lines such that all firms are bilaterally connected with each other, that is, by a tetrahedron. We formally derive this model in the next subsection.

Notably, there are other generalizations of the Hotelling duopoly toward n goods such as the *spokes model* by Chen and Riordan (2007). This model can be visualized by points on a circle that give firms' locations, and all firms' locations are connected by spokes. The brands are physically identical but are differentiated by their different locations. Consumers are uniformly

[6]In fact, the demand function (1) captures the Hoteling model if we have, as we have assumed, an inner solution, that is, if the price the marginal consumer pays plus her transportation costs do not exceed her valuation for the product.

distributed on the network of spokes, and each consumer has one most preferred brand (i.e., the firm that sits on the end of that spoke on which she sits), and unlike in this model there is not only one second preferred brand, but each of the other $N-1$ brands is equally likely to be her second preferred brand. While the model we build on is more tractable, in fact, the models are quite similar and nest special cases of each other (for a comparison see the discussion in Somaini and Einav 2013).

2.2 Derivation of the LCD-Model

The LCD-model can be derived from a horizontal product differentiation model in the spirit of the Hotelling duopoly model as suggested by Somaini and Einav (2013). There are $i = 1, ..., m$ firms each producing a horizontally differentiated product. The firms sell their goods in $j = 1, ..., n$ independent markets. In each market j, there are $l_m := \frac{m(m-1)}{2}$ Hotelling lines, such that all firms are directly linked with each other. The length of each line in market j is L^j. As in the Hotelling duopoly model, two firms i and i', with $i \neq i'$ are always located at the end points of a line. Let there be a total mass of consumers of M^j in market j, which is uniformly distributed over all lines. Thus, consumers are distributed with constant density $f^j := M^j/(l_m L^j)$ over each line of length L^j. Every consumer is distributed to one of the l_m lines and is identified by its address $x \in [0, L^j]$ on this line. All consumer have unit demands. A consumer x, located on a line connecting firms i and i', obtains utility of $U_i^j(x) = v^j - p_i^j - t^j|x_i - x|$ from consuming product i at price p_i and incurring "transportation" costs $t^j > 0$ per unit of distance, where x_i stands for firm i's location on the line. The parameter v^j stands for the gross utility of consuming one unit of the good a consumer obtains in market j.

Take the line between the firms i and i', with $i \neq i'$. Firm i's demand on the respective line is determined by the location of the indifferent consumer x' which follows from

$$U_i^j(x') = v^j - p_i^j - t^j x' = v^j - p_{i'}^j - t^j \left(L^j - x'\right) = U_{i'}^j(x'),$$

where we assumed that firm i is located at $x = 0$ and firm i' is located at $x = L^j$. Solving for x' we get the indifferent consumer and thus firm i's demand on the line connecting firms i and i':

$$D_{ii'}^j(p_i^j, p_{i'}^j) := x' f^j = \frac{1}{2}\left[L^j + \frac{1}{t^j}\left(p_{i'}^j - p_i^j\right)\right]\frac{2M^j}{m(m-1)L^j}.$$

The total demand of firm i in market j is then given by summing the "line-demands," $D^j_{ii'}(p^j_i, p^j_{i'})$, over all $i' \neq i$, which gives

$$D^j_i(p^j_1, ..., p^j_m) = \frac{M^j}{m} + \frac{M^j}{m(m-1)L^j t^j} \sum_{i' \neq i} \left(p^j_{i'} - p^j_i \right).$$

Thus, overall demand of firm i in market j follows from (1), with $a^j = \frac{M^j}{m}$ and $b^j = \frac{M^j}{m(m-1)L^j t^j}$.[7]

We finally note that the LCD-model can take care of loyal consumers, who always buy from one of the firms, if the price does not exceed their reservation prices. Suppose the mass of loyal consumers is K^j in market j, so that the total mass of consumers in market j becomes $M^j + K^j$. The mass of loyal consumers is equally distributed among the firms, so that every firm serves a mass of K^j/m of loyal consumers. Assume that a firm never wants to serve only its loyal consumers and that the loyals' reservation price is large enough, so that they are willing to buy at the price the indifferent consumers pay (for instance, it is v^j). In this scenario, firm i's demand is given by

$$D^j_i(p^j_1, ..., p^j_n) = \frac{M^j + K^j}{m} + \frac{M^j}{m(m-1)L^j t^j} \sum_{i' \neq i} \left(p^j_{i'} - p^j_i \right),$$

so that the demand of firm i in market j follows from (1), with $a^j = \frac{M^j + K^j}{m}$ and $b^j = \frac{M^j}{m(m-1)L^j t^j}$.

The economic interpretation of this demand model is that a certain product category (e.g., automobiles, breakfast cereals, detergence products or sports shoes) consists of m brands which are offered in all n (geographical) markets. In the context of the European Single market, we think of international brands offered in several EU countries. Demand in each market is differentiated and all brands are sold in all markets. Critically, consumer demands across markets differ with respect to the strength of consumer preferences for the different brands (i.e., their disutility per unit of distance in product space, t^j, differs).

3 Analysis

This LCD-model has several convenient properties that we list in the following.

[7] We assumed that the prices are such that consumers' are willing to buy at the posted prices; i.e., their gross utilities v^j are sufficiently large. In addition, we suppose that the utilities from buying are larger than their reservation utilities.

(A1) $\frac{\partial D_i^j}{\partial p_i^j} = -(m-1)b^j$ for all i and j.

(A2) $\sum_i D_i^j(p_1, ..., p_m) = ma^j$.

(A3) $D_i^j - D_{i'}^j = b^j m(p_{i'}^j - p_i^j)$.

As a consequence of these properties, firm i's demand is linear in its price (A1), aggregate demand is inelastic (A2), and the demand differences between two firms are pinned down by the difference in prices these two firms set and therefore independent from other prices charged (A3).

Throughout the paper we maintain the assumption that the discriminatory pricing equilibrium, $\{\overline{p}_i^j\}_{i=1,...,m}^{j=1,...,n}$, and the uniform pricing equilibrium, $\{\widehat{p}_i\}_{i=1,...,m}$, are unique and interior. Obviously, there exists a unique interior equilibrium both under discriminatory and under uniform pricing if costs are not too heterogenous (see also Somaini and Einav, 2013).

In the following proposition, we compare the Nash equilibrium when firms simultaneously charge uniform prices across markets and when firms engage in third-degree price discrimination, thereby charging different prices in the markets.

Proposition 1. *Assume an LCD-model and constant marginal production costs $c_i \geq 0$ for all $i = 1, ..., m$. Then, the following properties are fulfilled:*
i) All bilateral price differences are the same under discriminatory and uniform pricing, such that $\overline{p}_{i'}^j - \overline{p}_i^j = \widehat{p}_{i'} - \widehat{p}_i = \frac{m-1}{2m-1}(c_{i'} - c_i)$ holds for all i, i' and j.
ii) All firms' output levels in all markets are the same in the discriminatory and the uniform pricing equilibrium; i.e., $D_i^j(\overline{p}_1^j, ..., \overline{p}_m^j) = D_i^j(\widehat{p}_1, ..., \widehat{p}_m) = a^j + b^j \left(\frac{m-1}{2m-1}\right) \sum_{i' \neq i}(c_{i'} - c_i)$ for all i and j.

Proof. Under discriminatory pricing each firm i maximizes

$$\max_{p_i^1, ..., p_i^n \geq 0} \pi_i = \sum_{j=1}^{n} D_i^j(p_1^j, ..., p_m^j)(p_i^j - c_i).$$

The unique and interior Nash equilibrium prices $\{\overline{p}_i^j\}_{i=1,...,m}^{j=1,...,n}$ fulfill

$$\frac{\partial D_i^j}{\partial p_i^j}(\overline{p}_i^j - c_i) + D_i^j = 0 \text{ for all } i \text{ and all } j. \tag{2}$$

Fix some j and take two firms $i \neq i'$. The equilibrium price difference $\overline{p}_{i'}^{j} - \overline{p}_{i}^{j}$ follows from subtracting the first-order conditions $\frac{\partial \pi_{i'}}{\partial p_{i'}^{j}} = 0$ and $\frac{\partial \pi_{i}}{\partial p_{i}^{j}} = 0$, which gives

$$\frac{\partial D_{i'}^{j}}{\partial p_{i'}^{j}}(\overline{p}_{i'}^{j} - c_{i'}) - \frac{\partial D_{i}^{j}}{\partial p_{i}^{j}}(\overline{p}_{i}^{j} - c_{i}) + D_{i'}^{j} - D_{i}^{j} = 0.$$

Using (A1) and (A3) we get

$$\begin{aligned} -(m-1)b^{j}(\overline{p}_{i'}^{j} - \overline{p}_{i}^{j}) - b^{j}m(\overline{p}_{i'}^{j} - \overline{p}_{i}^{j}) &= -(m-1)b^{j}(c_{i'} - c_{i}) \text{ or} \\ \overline{p}_{i'}^{j} - \overline{p}_{i}^{j} &= \frac{m-1}{2m-1}(c_{i'} - c_{i}). \end{aligned} \tag{3}$$

Under uniform pricing each firm i maximizes

$$\max_{p_i \geq 0} \pi_i = \sum_{j=1}^{n} D_{i}^{j}(p_1, ..., p_m)(p_i - c_i).$$

The unique and interior Nash equilibrium prices $\{\widehat{p}_i\}_{i=1,\ldots,m}$ fulfill

$$\sum_{j=1}^{n} \left[\frac{\partial D_{i}^{j}}{\partial p_i}(\widehat{p}_i - c_i) + D_{i}^{j} \right] = 0 \text{ for all } i. \tag{4}$$

Take two firms $i \neq i'$. The equilibrium price difference $\widehat{p}_{i'} - \widehat{p}_i$ follows from subtracting the first-order conditions $\frac{\partial \pi_{i'}}{\partial p_{i'}} = 0$ and $\frac{\partial \pi_i}{\partial p_i} = 0$, which gives

$$\sum_{j=1}^{n} \frac{\partial D_{i}^{j}}{\partial p_i}(\widehat{p}_{i'} - c_{i'}) - \sum_{j=1}^{n} \frac{\partial D_{i}^{j}}{\partial p_i}(\widehat{p}_i - c_i) + \sum_{j=1}^{n} D_{i'}^{j} - \sum_{j=1}^{n} D_{i}^{j} = 0.$$

Using (A1) and (A3) we get

$$-(m-1)\sum_{j=1}^{n} b^{j}(\widehat{p}_{i'} - \widehat{p}_i) - m(\widehat{p}_{i'} - \widehat{p}_i)\sum_{j=1}^{n} b^{j} = -(m-1)\sum_{j=1}^{n} b^{j}(c_{i'} - c_i) \text{ or}$$

$$\widehat{p}_{i'} - \widehat{p}_i = \frac{m-1}{2m-1}(c_{i'} - c_i). \tag{5}$$

From (3) it follows that the price difference between two firms i and i' is the same in all markets j under discrimination. Comparison with (5) shows that the price difference under uniform pricing yields exactly the same difference. Finally, part ii) of the proposition follows from substituting (3) for all $i' \neq i$ into (1). **Q.E.D.**

Price competition yields the same price differences under discriminatory and uniform pricing (part i) of Proposition1). Consequently, firms' output levels in any market j are independent of the pricing regime (part ii) of Proposition 1).[8] In addition, when the number of firms increases, price differences approach marginal cost differences from below.[9] The underlying demand system ensures that price differences are fully driven by supply side features; namely, marginal cost asymmetries and the number of firms m.

Interestingly, even though price differences between the firms are always the same under discriminatory pricing in every market j, any firm i's market shares may differ across different markets. The market share of firm i in market j is given by

$$s_i^j := \frac{D_i^j}{\sum_{i=1}^m D_i^j} = \frac{1}{m}\left[1 + \frac{b^j}{a^j}\left(\frac{m-1}{2m-1}\right)\sum_{i' \neq i}(c_{i'} - c_i)\right],$$

where the last equality follows from (A2) and from part iii) of Proposition 1. Note also that $\sum_{i' \neq i}(c_{i'} - c_i) = m(c^e - c_i)$, with $c^e := \sum_{i=1}^m c_i/m$. Suppose $b^j/a^j > b^{j'}/a^{j'}$ holds. Then, $s_i^j > s_i^{j'}$ ($s_i^j < s_i^{j'}$) follows if and only if $c_i < c^e$ ($c_i > c^e$). A firm with below-average marginal cost, therefore, gets a larger market share in market j than in j', when the competitive intensity (as measured by b^j/a^j) increases.[10] This result also mirrors (A3), which states that the demand difference between two firms gets larger when the parameter b^j increases.

Proposition 1 implies that the difference of consumer surplus under uniform and discriminatory pricing, $\widehat{CS} - \overline{CS}$, which must be equal to the reversed difference of total profits, $\sum_i \overline{\pi}_i - \sum_i \widehat{\pi}_i$, can be derived directly from comparing the uniform and the discriminatory prices.

[8] In the following, we drop the arguments of D_i^j, which from now on stands for the equilibrium values $D_i^j(\overline{p}_1^j, ..., \overline{p}_m^j)$ or $D_i^j(\widehat{p}_1, ..., \widehat{p}_m)$.

[9] Under both pricing regimes, the price difference is equal to the marginal cost difference times the term $\frac{m-1}{2m-1}$, which increases monotonically in m over the interval $[1/3, 1/2)$. In the limiting case of $m \to \infty$ it approaches one. Thus, when the number of firms, m, increases, then bilateral price differences increase and approach marginal cost differences in the limit.

[10] In Section 2.2, we have shown how a^j and b^j can be derived from a generalized Hotelling model. In particular, b^j/a^j increases when the transportation costs parameter (t^j) decreases or the length of the Hotelling line (L^j) shortens. In Section 2.2 we have also considered a scenario with additional loyal consumers, in which case b^j/a^j decreases when the share of loyal consumers increases.

Corollary 1. *The difference of consumer surplus and the difference of total profits under uniform and discriminatory pricing are given by*

$$\widehat{CS} - \overline{CS} = \sum_{i=1}^{m} \overline{\pi}_i - \sum_{i=1}^{m} \widehat{\pi}_i = \sum_{i=1}^{m}\sum_{j=1}^{n} (\overline{p}_i^j - \widehat{p}_i) D_i^j.$$

Based on Proposition 1, we can easily calculate the Nash equilibrium prices under both pricing regimes. In the discriminatory regime, firm i's first-order condition in market j is given by (2). Solving for $\overline{p}_i^j$ we get

$$\overline{p}_i^j = c_i - \frac{D_i^j}{\frac{\partial D_i^j}{\partial p_i^j}} = c_i + \frac{a^j}{(m-1)b^j} + \left(\frac{1}{2m-1}\right) \sum_{i' \neq i} (c_{i'} - c_i). \tag{6}$$

Similarly, for the uniform pricing regime, the Nash equilibrium price of firm i can be obtained from firm i's first-order condition (4). Solving for $\widehat{p}_i$ we get

$$\widehat{p}_i = c_i - \frac{\sum_{j=1}^n D_i^j}{\sum_{j=1}^n \frac{\partial D_i^j}{\partial p_i}} = c_i + \frac{\sum_{j=1}^n \left[a^j + b^j \left(\frac{m-1}{2m-1}\right) \sum_{i' \neq i} (c_{i'} - c_i)\right]}{(m-1)\sum_{j=1}^n b^j}. \tag{7}$$

We next examine how the discriminatory and uniform pricing equilibrium are related. Define firm i's equilibrium price elasticity in market j under discriminatory pricing by

$$\overline{E}_i^j := E_i^j(\overline{p}_1^j, ..., \overline{p}_m^j) := -\frac{\partial D_i^j}{\partial p_i^j} \frac{\overline{p}_i^j}{D_i^j} \tag{8}$$

and firm i's aggregate equilibrium price elasticity under uniform pricing by

$$\widehat{E}_i := E_i(\widehat{p}_1, ..., \widehat{p}_m) := -\frac{\sum_{j=1}^n \frac{\partial D_i^j}{\partial p_i} \widehat{p}_i}{\sum_{j=1}^n D_i^j}. \tag{9}$$

Firm i's Lerner index under discriminatory pricing is equal to the weighted arithmetic mean of its market-specific Lerner indices, $\overline{L}_i^j := \frac{\overline{p}_i^j - c_i}{\overline{p}_i^j}$, where the weights are given by firm i's output in market j, D_i^j, relative to its total output, $\sum_{j=1}^n D_i^j$; i.e.,

$$\overline{L}_i := \sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^n D_i^j} \overline{L}_i^j\right].$$

Define the aggregate Lerner index under discriminatory pricing by $\overline{L} := \sum_i s_i \overline{L}_i$, where $s_i := \frac{\sum_j D_i^j}{\sum_j \sum_i D_i^j}$ stands for firm i's overall market share. In case of uniform pricing, $\widehat{L}_i := \frac{\widehat{p}_i - c_i}{\widehat{p}_i}$ and

$\widehat{L} := \sum_i s_i \widehat{L}_i$ stand for firm i's Lerner index and for the aggregate Lerner index, respectively. The following proposition then follows.

Proposition 2. *Assume an LCD-model. The comparison of the discriminatory and the uniform pricing equilibrium gives the following relations:*

i) Firm i's aggregate equilibrium price elasticity under uniform pricing is given by the weighted Arithmetic Mean Formula:

$$\widehat{E}_i = \sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^n D_i^j} \overline{E}_i^j \right] \text{ holds for all } i.$$

ii) Firm i's Lerner index under uniform pricing is given by the weighted Harmonic Mean Formula:

$$\widehat{L}_i = \frac{1}{\sum_{j=1}^n \left[\frac{D_i^j}{\sum_{j=1}^n D_i^j} \frac{1}{\overline{L}_i^j} \right]} \text{ holds for all } i.$$

iii) If firms are asymmetric (i.e., $c_i \neq c_i'$ with $i \neq i'$ for all i) and if all firms' marginal costs are strictly positive, then all firm-level Lerner indices and the aggregate Lerner index are strictly smaller under uniform pricing than under discriminatory pricing; i.e.,

$$\widehat{L}_i < \overline{L}_i \text{ holds for all } i \text{ and } \widehat{L} < \overline{L}.$$

iv) If firms are asymmetric (i.e., $c_i \neq c_i'$ with $i \neq i'$ for all i) and if all firms' marginal costs are strictly positive, then firm i's uniform price is strictly smaller than the weighted arithmetic mean of its discriminatory prices; i.e.

$$\widehat{p}_i < \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^n D_i^j} \overline{p}_i^j \text{ holds for all } i.$$

Proof. Assume discriminatory pricing. Summing up firm i's first-order conditions over all markets j gives

$$\sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} (\overline{p}_i^j - c_i) \right] + \sum_{j=1}^{n} D_i^j = 0.$$

Under uniform pricing, firm i's first-order condition is given (4). From Proposition 1 it follows that firm i's equilibrium demand is the same in every market under both pricing regimes, which implies

$$\sum_{j=1}^{n} D_i^j(\overline{p}_1^j, ..., \overline{p}_m^j) = \sum_{j=1}^{n} D_i^j(\widehat{p}_1, ..., \widehat{p}_m).$$

It thus follows that

$$\sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i}(\widehat{p}_i - c_i) = \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j}(\overline{p}_i^j - c_i) \right]. \tag{10}$$

Simplifying and expanding both sides we get

$$\frac{\sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i}\widehat{p}_i}{\sum_{j=1}^{n} D_i^j} \left(\sum_{j=1}^{n} D_i^j \right) = \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} \frac{\overline{p}_i^j}{D_i^j} D_i^j \right].$$

Using (8) and (9) we get

$$\begin{aligned} \widehat{E}_i \sum_{j=1}^{n} D_i^j &= \sum_{j=1}^{n} \left[\overline{E}_i^j D_i^j \right] \text{ or} \\ \widehat{E}_i &= \sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{E}_i^j \right]. \end{aligned} \tag{11}$$

The equilibrium aggregate demand elasticity under uniform pricing of firm i is equal to the weighted arithmetic mean of firm i's demand elasticities under discriminatory pricing. The weight of firm i's demand elasticity in market j is given by the share of firm i's total output sold in market j. This gives part i).

Next, we can re-write firm i's first-order condition under uniform pricing (see (4)) as

$$\frac{\widehat{p}_i - c_i}{\widehat{p}_i} = \frac{1}{\widehat{E}_i}.$$

Likewise, under discriminatory pricing we can re-write each of firm i's first-order conditions (see (2)) as

$$\frac{\overline{p}_i^j - c_i}{\overline{p}_i^j} = \frac{1}{\overline{E}_i^j}.$$

Taken together and using (11) we get

$$\frac{\widehat{p}_i - c_i}{\widehat{p}_i} = \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{E}_i^j \right]} = \frac{1}{\sum_{j=1}^{n} \left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \left(\frac{\overline{p}_i^j - c_i}{\overline{p}_i^j} \right)^{-1} \right]}. \tag{12}$$

Using the definitions of $\widehat{L}_i$ and $\overline{L}_i^j$, we get the formula stated in part ii) of the proposition. By

Jensen's inequality,[11] it must be that

$$\sum_{j=1}^{n}\left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\frac{1}{\overline{L}_i^j}\right] > \frac{1}{\sum_{j=1}^{n}\left[\frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\overline{L}_i^j\right]},$$

which implies $\widehat{L}_i < \overline{L}_i$ and also $\widehat{L} < \overline{L}$, because s_i is independent of the pricing regime. This proves part iii) of the proposition. Thus, part iii) follows from part ii).

Next we show that part iv) follows from part iii) (namely, $\widehat{L}_i < \overline{L}_i$) and is, therefore, also a consequence of the harmonic mean formula. Note first that we can re-write $\overline{L}_i$ as

$$\overline{L}_i = 1 - c_i \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\frac{1}{\overline{p}_i^j}.$$

Thus, $\widehat{L}_i < \overline{L}_i$ is equivalent to

$$\begin{aligned} \frac{\widehat{p}_i - c_i}{\widehat{p}_i} &< 1 - c_i \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\frac{1}{\overline{p}_i^j} \text{ or} \\ \frac{1}{\widehat{p}_i} &> \sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\frac{1}{\overline{p}_i^j}. \end{aligned} \tag{13}$$

By Jensen's Inequality, the right-hand side of (13) is strictly larger than the inverse of the weighted arithmetic mean of the discriminatory prices, so that

$$\frac{1}{\widehat{p}_i} > \frac{1}{\sum_{j=1}^{n} \frac{D_i^j}{\sum_{j=1}^{n} D_i^j}\overline{p}_i^j}$$

follows, from which we directly get the inequality stated in part iv) of the proposition. **Q.E.D.**

Proposition 2 generalizes Holmes' (1989) conjecture that average prices increase under discriminatory prices when compared with uniform pricing to an oligopoly with asymmetric firms. Holmes assumed symmetric firms and a constant elasticity demand at the firm level with inelastic market demand to show his conjecture. Relatedly, Armstrong (2007) has shown that this conjecture holds true for symmetric firms in a model closely related to ours, namely in a multi-market Hotelling model. Proposition 2 shows that his conjecture is also valid when firms are asymmetric and the underlying demand system ensures that market demands are inelastic.

[11] Jensen's inequality implies that for any positive random variable X with strictly positive expected value $E(X)$ the inequality $E\left[\frac{1}{X}\right] > \frac{1}{E(X)}$ holds.

According to part i) of Proposition 2, each firm's aggregate equilibrium elasticity under uniform pricing is the weighted arithmetic mean of a firm's equilibrium elasticities under discriminatory pricing, which follows from the fact that equilibrium quantities do not change with the pricing regime (Proposition 1). Part ii) shows that the Lerner index of any firm i under uniform pricing is the weighted harmonic mean of its market-specific Lerner indices under discriminatory pricing, where the weights are given by firm i's output in market j, relative to its total output. Part iii) states that all firms' Lerner indices and the aggregate Lerner index are lower under uniform pricing than under discriminatory pricing. This follows directly from part ii), because the (weighted) harmonic mean is always lower than the (weighted) arithmetic mean (unless all relative margins are equal). This relation gives a clear-cut assessment of the overall effect of uniform pricing on market power. Uniform pricing unambiguously constrains firms' market power, so that firms' ability to raise prices above marginal costs is smaller than under discriminatory pricing.

The harmonic mean formula implies that all firms' uniform prices are strictly smaller than the weighted arithmetic mean of their discriminatory prices (part iv) of Proposition 4). Using Corollary 1, we then know for sure that consumer surplus must be strictly larger under uniform pricing than under discriminatory pricing. This follows from noticing that

$$\widehat{CS} - \overline{CS} = \sum_{i=1}^{m} \sum_{j=1}^{n} (\overline{p}_i^j - \widehat{p}_i) D_i^j = \sum_{i=1}^{m} \left[\sum_{j=1}^{n} D_i^j \left(\sum_{j=1}^{n} \left(\frac{D_i^j}{\sum_{j=1}^{n} D_i^j} \overline{p}_i^j \right) - \widehat{p}_i \right) \right] > 0,$$

where the inequality follows from part iv) of Proposition 2. As all firms realize lower relative margins under uniform pricing according to the harmonic mean formula, it must be true that prices decrease on average which must increase consumer surplus and reduce total producer surplus accordingly. This is intuitive, as all output levels do not change under both pricing regimes.

We, finally, state our central result that the counterfactual uniform price as well as the consumer surplus gain from non-discriminatory prices can be calculated only based on market data under discriminatory pricing (i.e., prices and quantities).

Corollary 2. *Each firm's price under uniform pricing as well as the consumer surplus gain from uniform pricing can be calculated only based on market data under discriminatory pricing:*

$$\widehat{p}_i = \sum_{j=1}^{n} \left[\frac{b^j}{\sum_{j=1}^{n} b^j} \overline{p}_i^j \right].$$

Consumer surplus gain is given by

$$\widehat{CS} - \overline{CS} = \frac{1}{m-1} \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \frac{\left(D_i^j\right)^2}{b^j} - \frac{\left(\sum_{j=1}^{n} D_i^j\right)^2}{\sum_j b^j} \right),$$

where b^j can be determined from observables by (A3).

Proof. From (10) and (A1) we get

$$\begin{aligned} \sum_{j=1}^{n} \frac{\partial D_i^j}{\partial p_i} \widehat{p}_i &= \sum_{j=1}^{n} \left[\frac{\partial D_i^j}{\partial p_i^j} \overline{p}_i^j \right] \text{ or} \\ \widehat{p}_i &= \sum_{j=1}^{n} \left[\frac{b^j}{\sum_{j=1}^{n} b^j} \overline{p}_i^j \right]. \end{aligned} \tag{14}$$

Using (A3), which gives $b^j = \frac{D_i^j - D_{i'}^j}{m(p_{i'}^j - p_i^j)}$, we get $\widehat{p}_i$ directly from observed prices $\overline{p}_i^j$. Substituting (14) into the formula stated in Corollary 1, we get the consumer surplus gain from uniform pricing as stated above. **Q.E.D.**

Consumers as a whole are always better off when firms must charge a uniform price across markets. Correspondingly, every firm realizes a higher profit when all firms engage in price discrimination. From the firms' perspective, the discriminatory equilibrium Pareto-dominates the uniform pricing equilibrium. It follows that firms jointly have an incentive to coordinate market segmentation (e.g., by geo-blocking or, more generally, by restricting buyer arbitrage between markets). Thus, our results appear to be relevant for price discrimination along national markets (as in the EU).

Market–specific Delivery Costs We can easily include market-specific delivery costs $c^j \geq 0$ per unit of good for all j which affect all firms equally. In this case firm i's marginal cost of selling products in market j is given by $c_i + c^j$. Clearly, this does not affect the price differences in any market, so that all results of Proposition 1 remain valid. All equilibrium prices (6) and (7) as well as the arithmetic mean and the harmonic mean formulas of Proposition 2 also apply, but then at different marginal cost levels $c_i + c^j$ instead of c_i. Moreover, Corollary 2 likewise applies so that the formula for the consumer harm due to price discrimination is not affected by consideration of market-specific delivery costs.

4 Extension: Arbitrage Costs

We here show that the harmonic mean formula can be extended to take care of arbitrage costs. Assume that buyers can arbitrage among markets with arbitrage costs of $r \geq 0$ per unit. We focus on the case with $n, m = 2$. Thus discriminatory prices, $\{\overline{p}_i^j(r)\}_{i=1,2}^{j=1,2}$, must fulfill the requirement $\overline{p}_i^1 - \overline{p}_i^2 \leq r$ for $i = 1, 2$. Suppose that the constraints bind. The following proposition states the main features of the arbitrage-constrained third-degree price discrimination equilibrium.

Proposition 3. *Assume an LSC-model with $n, m = 2$. Assume $\overline{p}_i^1 > \widehat{p}_i > \overline{p}_i^2$. Suppose the arbitrage constraint is binding for both firms; i.e., $\overline{p}_i^1 - \overline{p}_i^2 \leq r$ for $i = 1, 2$. Then, the arbitrage-constrained Nash equilibrium prices $\{\overline{p}_i^j(r)\}_{i=1,2}^{j=1,2}$ are given by*

$$\overline{p}_i^1(r) = \widehat{p}_i + r\overline{\alpha} \text{ and } \overline{p}_i^2(r) = \widehat{p}_i - r(1 - \overline{\alpha}),$$

where $\overline{\alpha} := \frac{b^2}{b^1+b^2}$, with $\overline{\alpha} \in (0, 1)$. All price differences $p_{i'}^j - p_i^j$ and each firm's output in any market remains the same as under unconstrained discrimination or uniform pricing.

Proof. Each firm $i = 1, 2$ maximizes its profit $\pi_i = \sum_{j=1}^{2} \left[D_i^j (p_i^j(r) - c_i) \right]$ subject to $p_i^1(r) - p_i^2(r) \leq r$ for $i = 1, 2$. We obtain two first-order conditions of the constrained maximization problems:

$$\sum_{j=1}^{2} \left[\frac{\partial D_i^j}{\partial p_i^j} (p_i^j(r) - c_i) + D_i^j \right] = 0 \text{ with } \overline{p}_i^1(r) - \overline{p}_i^2(r) \leq r \text{ for } i = 1, 2. \tag{15}$$

Substitute $p_i^1(r) = \widehat{p}_i + \alpha r$ and $p_i^2(r) = \widehat{p}_i - (1 - \alpha)r$, with $\alpha \in [0, 1]$, so that $p_i^1 - p_i^2 = r$ holds for $i = 1, 2$. This gives

$$\frac{\partial D_i^1}{\partial p_i^1} (\widehat{p}_i + \alpha r - c_i) + D_i^1 + \frac{\partial D_i^2}{\partial p_i^2} (\widehat{p}_i - (1 - \alpha)r - c_i) + D_i^2 = 0 \text{ for } i = 1, 2. \tag{16}$$

or

$$\underbrace{\left[\left(\frac{\partial D_i^1}{\partial p_i^1}+\frac{\partial D_i^2}{\partial p_i^2}\right)(\widehat{p}_i-c_i)+D_i^1+D_i^2\right]}_{\textit{first term}}+r\underbrace{\left(\frac{\partial D_i^1}{\partial p_i^1}\alpha-\frac{\partial D_i^2}{\partial p_i^2}(1-\alpha)\right)}_{\textit{second term}}=0 \text{ for } i=1,2. \qquad (17)$$

Note that each firm's equilibrium output levels do not change under the proposed solution, because $p_{i'}^j(r)-p_i^j(r)=\widehat{p}_{i'}-\widehat{p}_i$ for all i, i' and j. Note that, for each firm i, the *first term* of (17) is equal to its first-order condition under uniform pricing (4). Thus, the *first term* in the first-order conditions of firms 1 and 2 is zero at $\{p_i^j(r)\}_{i=1,2}^{j=1,2}$. The *second term* is zero at

$$\overline{\alpha}=\frac{\frac{\partial D_i^2}{\partial p_i^2}}{\frac{\partial D_i^1}{\partial p_i^1}+\frac{\partial D_i^2}{\partial p_i^2}}=\frac{b^2}{b^1+b^2}.$$

Thus, $\widehat{p}_i^1(r)=\widehat{p}_i+\overline{\alpha}r$ and $\widehat{p}_i^2(r)=\widehat{p}_i-(1-\overline{\alpha})r$ solves the system of first-order conditions (15).

Q.E.D.

From Proposition 3 it follows that

$$\overline{\alpha}=\frac{\overline{p}_i^1-\widehat{p}_i}{\overline{p}_i^1-\overline{p}_i^2} \text{ and } 1-\overline{\alpha}=\frac{\widehat{p}_i-\overline{p}_i^2}{\overline{p}_i^1-\overline{p}_i^2},$$

so that a lower value of the arbitrage parameter r must decrease the average price $\frac{D_i^1}{D_i^1+D_i^2}\overline{p}_i^1(r)+\frac{D_i^2}{D_i^1+D_i^2}\overline{p}_i^2(r)$ and thus increases consumer surplus. In other words, any policy that makes cross-market arbitrage more effective is to the benefit of consumers as a whole.

5 Conclusion

In this paper, we analyzed the effects of oligopolistic third-degree price discrimination on consumer surplus. Under the assumption of full market coverage, consumer surplus is always lower, but firms' profits are always higher if price discrimination is feasible. We present a simple formula that allows to calculate the consumer surplus loss and firm profit gain of third-degree price discrimination solely based on observable market data under discriminatory pricing (prices and quantities). Notably, our analysis generalizes to the case where firms' do not only face different cost function, but where they also face different (delivery) costs for different markets.

We have used a fairly specific demand model (a Hotelling model of product differentiation generalized to an oligopoly with $m>2$ firms). Yet, that model exhibits certain "nice" features which should be also instructive for a more general analysis of oligopolistic price discrimination when firms' are asymmetric. First of all, firms' equilibrium outputs are the same under discriminatory and uniform pricing, which extends also to the case of arbitrage-constrained prices. It then follows that the Lerner index of each product under uniform pricing is given by the weighted harmonic mean of the market-specific Lerner indices under discriminatory pricing. Consequently, average prices are higher and aggregate consumer welfare is lower under discriminatory pricing.

It is conceivable that those clear-cut results no longer hold when price discrimination leads to quantity effects by either changing firms' market shares in a covered market and/or changing market demand when it is not inelastic. For instance, productive efficiency may increase under discrimination when more efficient firms tend to expand their outputs. Likewise, when firms' demands are non-linear, firms' market-specific demand elasticities may change in a way which is more favorable for discrimination, so that prices do not increase much in less competitive markets and decrease overproportionally in more competitive markets. But our results can be empirically tested against such alternative hypotheses: the harmonic mean formula predicts that price discrimination affects market prices (in absolute terms) the more the lower the competitive intensity of the market is (as measured by the market-specific degree of product differentiation).

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